2014 Combinatorics Workshop

Program and Abstract

Ajou University

October 31 – November 1, 2014, Suwon, Korea

Program

Date

October 31 (Fri) – November 1 (Sat), 2014

Venue

Paldal Hall 311, Ajou University, Suwon, Korea

Official webpage

http://ajou.ac.kr/∼schoi/Workshop/combinatorics2014

Program Committee

- Cho, Soojin (Ajou University)
- Choi, Suyoung (Ajou University)
- Park, Boram (Ajou University)

Advisory Committee

- Committee of Mathematics for Information Sciences, The Korean Mathematical Society (Chair: Suh-Ryung Kim, Seoul National University)

Sponsors

- National Research Foundation of Korea
- Ajou University

Speakers

Invited talks

- Woong Kook (Seoul National University)
- Sang June Lee (Duksung Women's University)
- Boram Park (Ajou University)
- Heesung Shin (Inha University)

Contributed talks

- Jihoon Choi (Seoul National University)
- Kwang Ju Choi (NIMS)
- Jisu Jeong (KAIST)
- JiYoon Jung (NIMS)
- Younjin Kim (KAIST)
- Hanchul Park (KIAS)
- Kyoungsuk Park (Ajou University)
- Seonjeong Park (NIMS)

Time table

October 31 (Friday)

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Invited talks

Topological invariants and tree numbers of matroids

Woong Kook

There are several simplicial complexes naturally associated with a matroid, and for that reason, topological methods are often employed in describing various matroid invariants.

In this talk, we will discuss topological invariants of the independent set complex and the broken circuit complex of a matroid, and their intriguing applications to the spectra of combinatorial Laplacians and the high-dimensional tree numbers of matroid complexes. If time permits, an application to Stanley's M-vector conjecture will be presented.

Name : Woong Kook
Address : Seoul National University
E-mail : woongkook@snu.ac.kr
Talk schedule : October 31 (Fri), 2014. 16:10–17:00

Universality of random graphs for graphs of maximum degree two

Sang June Lee

A graph $G = G(V, E)$ consists of a vertex set V and an edge set E of two element sets of V. For a positive integer n and a real number p with $0 \leq p \leq 1$, the random graph $G(n, p)$ is a graph obtained from the complete graph on n vertices by choosing each edge with probability p, independently of all other edges. After the random graph $G(n, p)$ was first introduced by Erdős in 1947, the theory of the random graph has become an active area of research.

One of the most interesting problems is the containment problem, in which one tries to obtain conditions on p for the property that $G(n, p)$ contains a given graph H as a subgraph with high probability.

In this talk, we consider a containment problem with respect to a family of graphs rather than a single graph. For a family $\mathcal{F}$ of graphs, a graph G is called *$\mathcal{F}$-universal* if G contains every graph in $\mathcal{F}$ as a subgraph. Constructing an universal graph with a small number of edges plays an important role in VLSI design in computer science, and is also an interesting combinatorics problem.

Let $\mathcal{F}^*$ be the family of all graphs on n vertices with maximum degree at most 2. We show that the random graph $G(n, p)$ is $\mathcal{F}^*$-universal with high probability provided $p \geq C\big(\frac{\log n}{n}\big)^{1/2}$ for a sufficiently large constant C.

This is joint work with Jeong Han Kim.

Name : Sang June Lee
Address : Duksung Women's University
E-mail : sanglee242@duksung.ac.kr
Talk schedule : November 1 (Sat), 2014. 9:00 – 09:50

Coloring powers of graphs

Boram Park

Coloring and List coloring of graphs not only are one famous research field in graph theory but also have many applications to real world problems. In this talk, we present several famous conjectures and results, which are related to coloring and list coloring of the power of a graph, including List Square Coloring Conjecture.

Name : Boram Park
Address : Ajou University
E-mail : borampark@ajou.ac.kr
Talk schedule : October 31 (Fri), 2014. 13:40 – 14:20

Continued fractions and generalized Fibonacci numbers

Heesung Shin

The *generalized Fibonacci numbers* G_n are defined by nonnegative integers G_0, G_1 and

$$G_n = G_{n-1} + G_{n-2} \quad \text{for } n \geq 2.$$

Thus $G_n = a\alpha^n + b\beta^n$ for some a and b, where $(\alpha, \beta) = (\frac{1+\sqrt{5}}{2}, \frac{1-\sqrt{5}}{2})$. The number G_n counts the ways to tile an $(n+1)$-board of dominoes and square tiles with phases for the initial tile. For example, the *Fibonacci numbers* $F_n = \frac{1}{\sqrt{5}}(\alpha^n - \beta^n)$ with $F_0 = 0$ and $F_1 = 1$ are generalized Fibonacci numbers and the *Lucas numbers* $L_n = \alpha^n + \beta^n$ with $L_0 = 2$ and $L_1 = 1$ are also. Thus F_n is the number of unphased $(n-1)$-tilings with squares and dominoes and L_n is the number of n-bracelets with square and dominoes.

Given positive integers $a_0, a_1, \ldots, a_n$, define $[a_0, a_1, \ldots, a_n]$ to be the continued fraction for

$$a_0 + \cfrac{1}{a_1 + \cfrac{1}{a_2 + \cfrac{1}{\ddots + \cfrac{1}{a_n}}}}$$

Let p and q be functions producing the numerator and denominator of the *finite continued fraction* $[a_0, a_1, \ldots, a_n]$ in lowest terms. For example, due to $[2, 3, 4, 2] = \frac{67}{29}$,

$$p(2, 3, 4, 2) = 67 \quad \text{and} \quad q(2, 3, 4, 2) = 29.$$

Here, let's do some combinatorics. The number $p(a_0, a_1, \ldots, a_n)$ counts the ways to tile an $(n+1)$-board of dominoes and *stackable* square tiles with height conditions $a_0, a_1, \ldots, a_n$. Also, since $q(a_0, a_1, \ldots, a_n) = p(a_1, \ldots, a_n)$, the number $q(a_0, a_1, \ldots, a_n)$ is the number of n-tilings with height conditions $a_1, \ldots, a_n$.

In this talk, we combinatorially prove that if m is odd, then

$$\frac{F_{(t+1)m}}{F_{tm}} = [L_m, L_m, \ldots, L_m]$$

where the number L_m appears t times. Also we could generalize it.

Name : Heesung Shin
Address : Inha University
E-mail : shin@inha.ac.kr
Talk schedule : November 1 (Sat), 2014. 10:50 – 11:40

Contributed talks

On competition graphs of n-tuply partial orders

Jihoon Choi

Studying competition graphs of interesting digraphs is a basic open problem in the study of competition graphs. In this context, Cho and Kim studied competition graphs of doubly partial orders and gave a nice characterization of the competition graphs of doubly partial orders. In this paper, we extend their results to a general case, which turns out to be quite interesting. Especially, we take a close look at the competition graphs of triply partial orders to present their meaningful properties.

Name : Jihoon Choi
Address : Seoul National University
E-mail : gaouls@naver.com
Talk schedule : October 31 (Fri), 2014. 14:55 – 15:20

The structure of non-planar graphs containing a critical edge

Kwang Ju Choi

By Kuratowski and Wagner's theorem, if we delete or contract any edge of $K_{3,3}$ or K_5, it becomes planar. We consider the class $\mathcal{C}$ of non-planar graphs G containing an edge e such that both $G \setminus e$ and G/e are planar. Then, every member of $\mathcal{C}$ has alternating double wheel as a topological minor. This is a joint work with Guoli Ding.

Name : Kwang Ju Choi
Address : NIMS
E-mail : kchoi@nims.re.kr
Talk schedule : October 31 (Fri), 2014. 15:20 – 15:45

Planar graphs with girth 5 are $(1, 10)$-colorable

Jisu Jeong

A graph is $(d_1, \ldots, d_r)$-colorable if its vertex set can be partitioned into r sets $V_1, \ldots, V_r$ so that the maximum degree of the graph induced by V_i is at most d_i for each $i \in \{1, \ldots, r\}$. For a given pair (g, d_1), the question of determining the minimum $d_2 = d_2(g, d_1)$ such that planar graphs with girth g are (d_1, d_2)-colorable has attracted much interest. The finiteness of $d_2(g, d_1)$ was known for all cases except when $(g, d_1) = (5, 1)$. Montassier and Ochem explicitly asked if $d_2(5, 1)$ is finite. We answer this question in the affirmative with $d_2(5, 1) \leq 10$; namely, we prove that all planar graphs with girth at least 5 are $(1, 10)$-colorable. Moreover, we prove that for any surface S of Euler genus γ, there exists $K = K(\gamma)$ where graphs with girth at least 5 that are embeddable on S are $(1, K)$-colorable. On the other hand, it was already known that there is no finite k where planar graphs with girth at least 5 are $(0, k)$-colorable. This is joint work with Hojin Choi, Ilkyoo Choi, and Geewon Suh.

Name : Jisu Jeong
Address : KAIST
E-mail : jjisu@kaist.ac.kr
Talk schedule : October 31 (Fri), 2014. 14:30 – 14:55

Descent pattern avoidance

JiYoon Jung

We extend the notion of consecutive pattern avoidance to considering sums over all permutations where each term is a product of weights depending on each consecutive pattern of a fixed length. We study the problem of finding the asymptotics of these sums. Our technique is to extend the spectral method of Ehrenborg, Kitaev and Perry. When the weight depends on the descent pattern we show how to find the equation determining the spectrum. We give two length 4 applications, and we show that the error term in the asymptotic expression for a weighted pattern of length 3 is the smallest possible.

Name : JiYoon Jung
Address : NIMS
E-mail : jjung328@gmail.com
Talk schedule : November 1 (Sat), 2014. 11:50 – 12:15

On the Erdös-Ko-Rado Theorem and the Bollobas Theorem for t-intersecting families

Younjin Kim

A family $\mathcal{F}$ of sets is t - intersecting if any two members of $\mathcal{F}$ have at least t common elements. In 1961, Erdös, Ko, and Rado proved that the maximum size of a t-intersecting family $\mathcal{F} \subset \binom{[n]}{\le k}$ is $\sum_{i=0}^{k-t} \binom{n-t}{k-t-i}$ if $n \ge n_0(k,t)$ for some $n_0(k,t)$. In 2014, Alon, Aydinian, and Huang considered families generalizing intersecting families, and gave a strengthening of this non-uniform EKR theorem when $t = 1$. In this paper, we give a strengthening of this theorem by considering families generalizing t-intersecting families for all $t \ge 1$. In 2004, Talbot generalized Bollobas's Two Families Theorem to t-intersecting families. In this paper, we proved a slight generalization of Talbot's result by using the probabilistic method. This is joint work with Dong Yeap Kang and Jaehoon Kim.

Name : Younjin Kim
Address : KAIST
E-mail : younjin@kaist.ac.kr
Talk schedule : November 1 (Sat), 2014. 10:00 – 10:25

Simplicial spheres, wedge operations and classification of toric varieties

Hanchul Park

A fundamental result in toric geometry is that there exists a bijection between toric varieties and fans. More generally, it is known that some class of manifolds having well-behaved torus actions, called topological toric manifolds. Similar to toric varieties, topological toric manifolds M can be described in terms of combinatorial data containing simplicial complexes K.

The classification problem for toric varieties is important but seems difficult even for smooth case. Smooth complete toric varieties of Picard number 3 were classified by Batyrev. In this talk, we discuss the classification of smooth complete toric varieties of Picard number 4. Furthermore, we also discuss about the classification of topological toric manifolds. For this, we use the relationship between M and simplicial wedge operation of K.

This is a joint work with Suyoung Choi.

Name : Hanchul Park
Address : KIAS
E-mail : hpark@kias.re.kr
Talk schedule : October 31 (Fri), 2014. 17:10 – 17:35

Permutation statistics and weak Bruhat order in permutation tableaux of type B

Kyoungsuk Park

Many important statistics of signed permutations are realized in the corresponding permutation tableaux or bare tableaux of type B: Alignments, crossings and inversions of signed permutations are realized in the corresponding permutation tableaux of type B, and the cycles of signed permutations are understood in the corresponding bare tableaux of type B. This leads us to relate the number of alignments and crossings with other statistics of signed permutations and also to characterize the covering relation in weak Bruhat order on Coxeter system of type B in terms of permutation tableaux of type B.

Name : Kyoungsuk Park
Address : Ajou University
E-mail : bluemk00@ajou.ac.kr
Talk schedule : November 1 (Sat), 2014. 12:15 – 12:40

Origami templates for simple polytopes

Seonjeong Park

An origami template is a collection of Delzant polytopes with some additional gluing data encoded by a template graph. In this talk, we define a smooth $\mathbb{Z}^n$-coloring on a simple polytope, and then we discuss the existence of an origami template for a given simple polytope with smooth $\mathbb{Z}^n$-coloring.

Name : Seonjeong Park
Address : NIMS
E-mail : seonjeong1124@nims.re.kr
Talk schedule : October 31 (Fri), 2014. 17:35 – 18:00

Memo